B.A. /B.Sc. Part-III (Honours) Special Examination, 2020 (1+1+1)
Subject: Mathematics
Paper: VII

Time: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any 4 questions: **4×5=20**

(a) Show that the distribution function $F(x)$ of a random variable X is monotonic non-decreasing function.

(b) The random variable X is distributed uniformly over the interval (0,2). Find the distribution function of the larger root of the quadratic equation $t^2 + 2t - X = 0$.

(c) Find the moment generating function of Binomial (n,p) distribution. Hence find its mean and variance.

(d) The joint probability density function of the random variables (X,Y) is given by

$$f(x,y) = k(3x+y),\ 1 \le x \le 3, 0 \le y \le 2$$

$$= 0 \qquad , \text{elsewhere}$$

Find the value of k and calculate $P(X+Y<2)$.

(e) If $\sigma_x = \sigma_y = 1$ and $\rho(aX+bY, bX+aY) = \frac{1+2ab}{a^2+b^2}$, find $\rho(X,Y)$.

(f) Using Tchebycheff's inequality show that in 1000 throws of unbiased coin the probability that the number of heads lies between 450 and 550 is at least $\frac{9}{10}$.

2. Answer any two questions: **2×5 = 10**

(a) Show that the sample mean $\bar{x}$ is asymptotically normal $(m, \frac{\sigma}{\sqrt{n}})$, where m is the population mean and σ is the population standard deviation.

(b) Show that the sample variance $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is not an unbiased estimator of σ^2. Find the unbiased estimator of σ^2.

(c) Find the maximum likelihood estimator of p for the observations x from the discrete distributions $f(x) = (1-p)^{x-1}p,\ \ x = 1, 2, 3 \ldots$

(d) For a normal (m, σ) population, find the interval estimation for m on the basis of a sample of size n from the population, when σ is known.

3. Answer any four questions: **4×5 = 20**

(a) Solve the following LPP graphically:

Maximize $z = 5x_1 + 7x_2$

subject to $3x_1 + 8x_2 \le 12,\ x_1 + x_2 \le 2,\ 2x_1 \le 3$ and $x_1, x_2 \ge 0$

(b) Show that the set of all feasible solutions of a linear programming problem is a convex set.

(c) Solve the following LPP by simplex method:

Maximize $z = 3x_1 + x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \le 2,\ x_1 + 2x_2 + 3x_3 \le 5,\ 2x_1 + 2x_2 + x_3 \le 6$ and $x_1, x_2, x_3 \ge 0$.

(d) Find initial basic feasible solution of the following transportation problem by Vogel's approximation method.

	D_1	D_2	D_3	Supply
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
Demand	50	80	80	

(e) Show that every two-person zero sum game can be converted to an L.P.P.

(f) Using dominance property, reduce the following pay off matrix and hence find the optimal strategies and the value of the game.

Player A

	2	2	1	-2	-3
Player B	4	3	4	-2	0
	5	1	2	5	6