B.A. /B.Sc. Part-III (Honours) Special Examination, 2020 (1+1+1)
Subject: Mathematics
Paper: VI

Time: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer *any four* questions: **4×5=20**

(a) A uniform rod of length $2a$ is placed with one end in contact with a horizontal table and is at an inclination α to the horizon and is allowed to fall. When it becomes horizontal, show that its angular velocity is $\sqrt{\frac{3g}{2a}\sin\alpha}$, whether the plane be perfectly smooth and perfectly rough.

(b) A cylinder rolls down a smooth plane whose inclination to the horizontal is α, as it goes; a fine string fixed to the highest point of the plane, find its acceleration and the tension of the string.

(c) Four equal heavy uniform rods are freely jointed to form a rhombus ABCD, which is freely suspended from A and is kept in shape of a square by an inextensible string connecting A and C. Show that the tension in the string is 2W where W is the weight of each rod.

(d) Show that the total kinetic energy of a system of two particles is equal to the sum of the kinetic energy of the system with respect to the centre of mass and the kinetic energy of the centre of mass.

(e) Explain how to determine the resultant thrust on a curved surface bounded by a plane curve exposed to pressure of heavy fluid at rest under gravity.

(f) A square lamina is placed vertically in a liquid of double its density. Prove that it can rest only with an edge or diagonal vertical.

2. Answer *any three* questions: **3×10 = 30**

(a) (i) Show that the kinetic energy of a rigid body rotating about a fixed point O is given by

$T = \frac{1}{2}\left(A\omega_1^2 + B\omega_2^2 + C\omega_3^2\right)$, where A, B, C are the principal moments of inertia at O and

ω_1, ω_2, ω_3 are the components of angular velocity along the principal axes respectively.

(ii) An elliptical lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation, prove that the eccentricity is 1/2. 5+5

(b) If the moments and products of inertia of a body about three perpendicular and concurrent axes are known, then find out the moment of inertia about any line passing through their meeting point. Also, as a particular case, find the result for a plane lamina. 8+2

(c) A perfectly rough body rests in equilibrium on a fixed body of convex upward. Discuss the conditions of stable and unstable equilibrium.

(d) A homogeneous liquid is rotating uniformly about a vertical axis under a given system of forces. Derive the differential equation for the pressure. Show that, if the gravity is the only external force, the free surface of the liquid is the paraboloid of revolution.
If the fluid is heterogeneous, show that the same paraboloids of revolution are also the surfaces of equi-density. 5+5

(e) State first law of thermodynamics. Derive a relation between pressure and volume in an adiabatic change.
If the pressure of the air varies as the $\left(1+\frac{1}{m}\right)$th power of the density, show that, the height of the atmosphere would be equal to $(m+1)$ times the height of the homogeneous atmosphere, neglecting variation of temperature and gravity. 2+3+5